International Research Journal of Education and Technology

Peer Reviewed Journal, ISSN 2581-7795

# A Nano topological Approach to Stability Analysis of the Human Digestive System

**R. Maheswari[1], V.Jaya Lakshmi[2]**

[1] Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College,

Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu.

[2] PG Scholar, Department of Mathematics, Shrimati Indira Gandhi College,

Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu.

## Abstract:

This study applies Nano topological concepts to model the human digestive system as a finite directed graph. Each organ is represented as a vertex and their functional relationships as edges. Using lower and upper approximations derived from neighbourhood structures, a mathematical measure called the Digestive Stability Index (DSI) is defined to evaluate subsystem stability. Two frameworks—General Nano topology and Initial Left Neighbourhood structure are used to analyze organ interactions and stability behavior within the system.

## Keywords:

Nano topology, Digestive Stability Index, Initial left neighbourhood,Upper and Lower Approximation

## 1.Introduction

Pawalak [11] introduced mathematical rough set theory in the early 1980's.In 2013, Lellis Thivagar introduced the concept of nano-topological spaces [3]. Many authors have employed Nano-topology in many different kinds of applications [1,2,4,5 and 6]. The notion of ideal Nano-topological space was introduced [7,8,9,10,12,13 and 14].In [15] Generating and discussions topological graph by three methods. On the other hand, it also performed comparisons and introduced independence and dependence of two graphs through a new scale. In [16] Applied both graph and topology on some of medical application such as the blood circulation in the human heart.

The human digestive system is a sophisticated biological network where the collective function of organs ensures the maintenance of life. Mathematically, this system can be viewed as a coordinated structure where stability is derived from the interaction between various components .To analyze the structural integrity of this system, we employ **Nano topology**, a branch of mathematics that uses lower and upper approximations to classify objects within a finite universe. A key contribution of this work is the development of the **Digestive Stability Index (DSI)**, a quantitative measure used to determine the stability of individual organs and their combinations. By comparing General Nanotopology with an Initial Left Neighbourhood structure, this paper examines how different mathematical perspectives affect the stability profile of biological systems

## 2.Preliminaries

**Definition 2.1**

**Graph**

A **Graph** $G = (V, E)$ is defined as a pair consisting of a non-empty, finite set of vertices(V) and a set of edges (E) representing unordered pairs of distinct vertices.

**Definition 2.2**

**Lower approximation**:

The **Lower approximation** of X with respect to ℝ is the set of all objects which can for certain classified as X with respect to ℝ and it is denoted by $\underline{\mathcal{L}}(x)$

$$(\boldsymbol{i.e})\underline{\mathcal{L}}(X) = \{\Re(x): \Re(x) \subseteq X\}$$

**Definition 2.3**

**Upper approximation:**

The **Upper approximation** of X with respect to ℝ is the set of all objects which can be possibly classified as X ⊆U with respect to ℝ and it is denoted by $\overline{\mathfrak{U}}(x)$.

$$(i.e)\,\overline{\mathfrak{U}}(X) = \{\mathfrak{R}(x) \cap X \neq \emptyset\}.$$

**Definition 2.4**

**Nano topology:**

Let $\mathbb{U}$ be a non-empty,finite universe of objects and ℝ be an equivalence relation on $\mathbb{U}$. Let $X \subseteq \mathbb{U}$. Let $\tau(X) = \{\mathbb{U}, \emptyset, \underline{\mathcal{L}}(X), \overline{\mathfrak{U}}(X), \mathfrak{B}nd(X)\}$. Then $\tau(X)$ is a topology on $\mathbb{U}$, called as the **Nano topology** with respect to X. Elements of the Nano topology are known as the Nano open sets in $\mathbb{U}$and $(\mathbb{U}, \tau(X))$ is called the Nano topological space.

**Definition 2.5**

**Initial left neighbourhood**

Let $\lambda \epsilon \mathbb{U}$.Then the **Initial left neighbourhood** of $\lambda$, is defined by

$\mathfrak{R}_{il}(\lambda) = \{\mu \in \mathbb{U} | \mathfrak{R}_l(\lambda) \subseteq \mathfrak{R}_l(\mu)\}$, where $\mathfrak{R}_l(\lambda) = \{\mu \in \mathbb{U} | \mu \mathfrak{R} \lambda\}$

**Definition 2.6**

**Initial left lower approximation**

The **Initial left lower approximation** of X with respect to ℝ is the set of all objects which can for certain be classified as $X \subseteq \mathbb{U}$ with respect to ℝ and it is denoted by $\underline{\mathcal{L}}_{il}(X)$

(i.e.) $\underline{\mathcal{L}}_{il}(X) = \{a \in \mathbb{U}: \mathfrak{R}_{il}(a) \subseteq X\}$.

**Definition 2.7**

**Initial left upper approximation**

The **Initial left upper approximation** of X with respect to ℝ is the set of all objects which can be possibly classified as $X \subseteq \mathbb{U}$ with respect to ℝ and it is denoted by $\underline{\mathfrak{U}}_{il}(X)$

(i.e.) $\underline{\mathfrak{U}}_{il}(X) = \{a \in \mathbb{U}: \mathfrak{R}_{il}(a) \cap X \neq \emptyset\}$

**Definition 2.8**

**Digestive Stability Index**

Let S ⊆ V.The Digestive Stability Index (DSI) of S is defined as

$$\mathrm{DSI(S)} = \frac{\left|\underline{\mathcal{L}}(\mathrm{S})\right|}{\left|\overline{\mathfrak{U}}(\mathrm{S})\right|}$$

Where $\left|\overline{\mathfrak{U}}(\mathrm{S})\right| \neq 0$, $\left|\underline{\mathcal{L}}(\mathrm{S})\right|$ denotes the cardinality of the lower approximation and $\left|\overline{\mathfrak{U}}(\mathrm{S})\right|$ denotes the cardinality of the upper approximation.

**Interpretation of DSI**

If $\mathrm{DSI}(S) = 0$, the subsystem is completely unstable , if $0 < DSI(S) < 1$, the subsystem is partially stable and if $\mathrm{DSI(S)} = 1$, the subsystem is fully stable.

## 3.Algorithm

**Step 1**. Define the vertex set V representing all digestive organs.

**Step 2**. Define the edge set E representing functional relationships among organs.

**Step 3.** Apply **Framework 1 (General Nano topology)** in which determine neighbourhood of each vertex and compute lower and upper approximations of S then calculate the Digestive Stability Index.

**Step 4.** Apply **Framework 2 (Initial Left Neighbourhood)** in which determine initial left neighbourhoods and compute lower and upper approximations of S then calculate the Digestive Stability Index.

**Step 5.** Compare framework results and analyze differences in stability evaluation.

**Step 6**. Interpret DSI values.

**Step 7**. Repeat Steps 3–6 for different subsystems S

**Step 8**. Record the DSI values obtained for each subsystem under both frameworks.

**Step 9**. Arrange the results in a comparison table .

**Step 10**. Conclude the stability characteristics of the digestive system based on comparative analysis..

## 4. Digestive Stability Index (DSI)

Let $G = (V, E)$ be a directed graph representing the digestive system.To study this mathematically, the digestive system is modeled as a finite set of vertices, where each vertex represents a digestive organ. The stability of individual organs and organ combinations is evaluated using the concept of Nano topology, which is based on lower and upper approximations derived from neighbourhood structures.

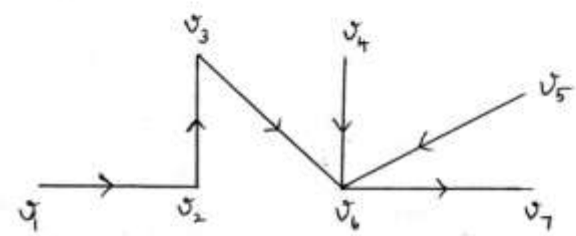


Figure 3.1

In this work, the digestive system is represented as a finite vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$

Where $v_1$: Mouth ; $v_2$: Esophagus ;$v_3$: Stomach ;$v_4$: Liver ;$v_5$: Pancreas ;$v_6$: Small Intestine; $v_7$: Large Intestine

The Edges can be represented as

$v_1 \to v_2$;$v_2 \to v_3$;$v_3 \to v_6$;$v_4 \to v_6$;$v_5 \to v_6$;$v_6 \to v_7$

The system is analyzed under the following two frameworks:

## Framework 1 : General Nanotopology

The primary objective is to construct a Digestive Stability Index (DSI) that quantitatively measures organ interdependence within these structures.

$N(v_1) = \{v_1\}; N(v_2) = \{v_1, v_2\}; N(v_3) = \{v_2, v_3\}; N(v_4) = \{v_4\};$

$N(v_5) = \{v_5\}; N(v_6) = \{v_3, v_4, v_5, v_6\}; N(v_7) = \{v_6, v_7\}$

**Step 1:**

Consider the case where only the mouth functions, represented by $\mathbf{S = v_1(Mouth)}$

The Corresponding approximations are given below

$$\underline{\mathcal{L}}(S) = \emptyset \Longrightarrow |\underline{\mathcal{L}}(S)| = 0; \overline{\mathfrak{U}}(S) = v_1 \Longrightarrow |\overline{\mathfrak{U}}(S)| = 1$$

The Digestive Stability Index is given by $DSI(S) = \frac{0}{1} = 0$

**Step 2:**

Consider the case where only the Stomach functions, represented by $\mathbf{S = v_3(Stomach)}$

The Corresponding approximations are given below

$$\underline{\mathcal{L}}(S) = \emptyset \Longrightarrow |\underline{\mathcal{L}}(S)| = 0; \overline{\mathfrak{U}}(S) = \{v_2, v_3, v_6\} \Longrightarrow |\overline{\mathfrak{U}}(S)| = 3$$

The Digestive Stability Index is given by $DSI(S) = \frac{0}{3} = 0$

**Step 3:**

Consider the case where only the Liver functions, represented by $\mathbf{S = v_4(Liver)}$

The Corresponding approximations are given below

$$\underline{\mathcal{L}}(S) = \emptyset \Longrightarrow |\underline{\mathcal{L}}(S)| = 0; \overline{\mathfrak{U}}(S) = \{v_4, v_6\} \Longrightarrow |\overline{\mathfrak{U}}(S)| = 2$$

The Digestive Stability Index is given by $DSI(S) = \frac{0}{2} = 0$

**Step 4:**

Consider the case where only the Small intestine functions, represented by $\mathbf{S} = \mathbf{v_6}(\mathbf{SmallIntestine})$

The Corresponding approximations are given below

$$\underline{\mathcal{L}}(\mathrm{S}) = \emptyset \Longrightarrow |\underline{\mathcal{L}}(\mathrm{S})| = 0; \overline{\mathfrak{U}}(\mathrm{S}) = \{v_3, v_4, v_5, v_6, v_7\} \Longrightarrow |\overline{\mathfrak{U}}(\mathrm{S})| = 5$$

The Digestive Stability Index is given by $\mathrm{DSI}(\mathrm{S}) = \frac{0}{5} = 0$

**Step 5:**

Consider the case where only the Liver and pancreas functions, represented by $\mathbf{S} = \mathbf{v_4}, \mathbf{v_5}(\mathbf{Liver} + \mathbf{pancreas})$

The Corresponding approximations are given below

$$\underline{\mathcal{L}}(\mathrm{S}) = \{v_4, v_5\} \Longrightarrow |\underline{\mathcal{L}}(\mathrm{S})| = 2; \overline{\mathfrak{U}}(\mathrm{S}) = \{v_4, v_5, v_6\} \Longrightarrow |\overline{\mathfrak{U}}(\mathrm{S})| = 3$$

The Digestive Stability Index is given by $\mathrm{DSI}(\mathrm{S}) = \frac{2}{3} = 0.67$

**Step 6:**

Consider the case where only the Stomach and small intestine functions, represented by $\mathbf{S} = \mathbf{v_3}, \mathbf{v_6}(\mathbf{Stomach}, \mathbf{Small\ intestine})$

The Corresponding approximations are given below

$$\underline{\mathcal{L}}(\mathrm{S}) = \emptyset \Longrightarrow |\underline{\mathcal{L}}(\mathrm{S})| = 0; \overline{\mathfrak{U}}(\mathrm{S}) = \{v_2, v_3, v_4, v_5, v_6, v_7\} \Longrightarrow |\overline{\mathfrak{U}}(\mathrm{S})| = 6$$

The Digestive Stability Index is given by $\mathrm{DSI}(\mathrm{S}) = \frac{0}{6} = 0$

**Step 7:**

Consider the case where the entire digestive system functions,represented by $\mathbf{S} = \mathbf{V}(\mathbf{G})(\mathbf{WholeDigestiveSystem})$

The Corresponding approximations are given below

$$\underline{\mathcal{L}}(\mathrm{S}) = \mathrm{V}(\mathrm{G}) \Longrightarrow |\underline{\mathcal{L}}(\mathrm{S})| = 7; \overline{\mathfrak{U}}(\mathrm{S}) = \mathrm{V}(\mathrm{G}) \Longrightarrow |\overline{\mathfrak{U}}(\mathrm{S})| = 7$$

The Digestive Stability Index is given by $\mathrm{DSI}(\mathrm{S}) = \frac{7}{7} = 1$

## Framework 2 : Initial Left Neighbourhood Structure

The following Left neighbourhoods are

$N_l(v_1) = \emptyset; N_l(v_2) = \{v_1\}; N_l(v_3) = \{v_2\}; N_l(v_4) = \emptyset; N_l(v_5) = \emptyset; N_l(v_6) = \{v_3, v_4, v_6\}; N_l(v_7) = \{v_6\}$

The corresponding Initial left neighbourhoods are given by

$N_{il}(v_1) = U; N_{il}(v_2) = v_2; N_{il}(v_3) = v_3; N_{il}(v_4) = U; N_{il}(v_5) = U; N_{il}(v_6) = v_6; N_{il}(v_7) = v_7$

Using this Neighbourhood system, approximations are recalculated.

**Step 1:**

Consider the case where only the mouth functions, represented by $\mathbf{S = v_1 (Mouth)}$

The Corresponding approximations are given below

$$\underline{\mathcal{L}_{il}}(S) = \emptyset \Longrightarrow \left|\underline{\mathcal{L}_{il}}(S)\right| = 0; \overline{\mathfrak{U}_{il}}(S) = \{v_1, v_4, v_6\} \Longrightarrow |\overline{\mathfrak{U}_{il}}(S)| = 3$$

The Digestive Stability Index is given by $DSI(S) = \frac{0}{3} = 0$

**Step 2:**

Consider the case where only the Stomach functions, represented by $\mathbf{S = v_3 (Stomach)}$

The Corresponding approximations are given below

$$\underline{\mathcal{L}_{il}}(S) = \{v_3\} \Longrightarrow \left|\underline{\mathcal{L}_{il}}(S)\right| = 1; \overline{\mathfrak{U}_{il}}(S) = \{v_1, v_3, v_4, v_6\} \Longrightarrow |\overline{\mathfrak{U}_{il}}(S)| = 4$$

The Digestive Stability Index is given by $DSI(S) = \frac{1}{4} = 0.25$

**Step 3:**

Consider the case where only the Liver functions, represented by $\mathbf{S = v_4 (Liver)}$

$$\underline{\mathcal{L}_{il}}(S) = \emptyset \Longrightarrow \left|\underline{\mathcal{L}_{il}}(S)\right| = 0; \overline{\mathfrak{U}_{il}}(S) = \{v_1, v_4, v_5\} \Longrightarrow |\overline{\mathfrak{U}_{il}}(S)| = 3$$



 

The Digestive Stability Index is given by $DSI(S) = \frac{0}{3} = 0$

**Step 4:**

Consider the case where only the Small intestine functions, represented by $\mathbf{S} = \mathbf{v_6}(\mathbf{SmallIntestine})$

The Corresponding approximations are given below

$$\underline{\mathcal{L}_{il}}(S) = \{v_6\} \Longrightarrow \left|\underline{\mathcal{L}_{il}}(S)\right| = 1; \overline{\mathfrak{U}_{il}}(S) = \{v_1, v_4, v_5, v_6\} \Longrightarrow |\overline{\mathfrak{U}_{il}}(S)| = 4$$

The Digestive Stability Index is given by $DSI(S) = \frac{1}{4} = 0.25$

**Step 5:**

Consider the case where only the Liver and pancreas functions, represented by $\mathbf{S} = \mathbf{v_4}, \mathbf{v_5}(\mathbf{Liver} + \mathbf{pancreas})$

The Corresponding approximations are given below

$$\underline{\mathcal{L}_{il}}(S) = \emptyset \Longrightarrow \left|\underline{\mathcal{L}_{il}}(S)\right| = 0; \overline{\mathfrak{U}_{il}}(S) = \{v_1, v_4, v_5\} \Longrightarrow |\overline{\mathfrak{U}_{il}}(S)| = 3$$

The Digestive Stability Index is given by $DSI(S) = 0$

**Step 6:**

Consider the case where only the Stomach and small intestine functions, represented by $\mathbf{S} = \mathbf{v_3}, \mathbf{v_6}(\mathbf{Stomach}, \mathbf{Small\ intestine})$

The Corresponding approximations are given below

$$\underline{\mathcal{L}_{il}}(S) = \{v_3, v_6\} \Longrightarrow \left|\underline{\mathcal{L}_{il}}(S)\right| = 2; \overline{\mathfrak{U}_{il}}(S) = \{v_1, v_3, v_4, v_5, v_6\} \Longrightarrow |\overline{\mathfrak{U}_{il}}(S)| = 5$$

The Digestive Stability Index is given by $DSI(S) = \frac{2}{5} = 0.4$

**Step 7:**

Consider the case where the entire digestive system functions,represented by $\mathbf{S} = \mathbf{V}(\mathbf{G})(\mathbf{WholeDigestiveSystem})$

The Corresponding approximations are given below



 

$$\underline{\mathcal{L}}_{il}(S) = V(G) \Longrightarrow \left|\underline{\mathcal{L}}_{il}(S)\right| = 7; \overline{\mathfrak{U}}_{il}(S) = V(G) \Longrightarrow |\overline{\mathfrak{U}}_{il}(S)| = 7$$

The Digestive Stability Index is given by $DSI(S) = \frac{7}{7} = 1$

## 5. Comparative Framework

This table shows a comparison of the Digestive Stability Index (DSI) values for different digestive organs using two frameworks

**Table 4.1**

| Organs | DSI(Nanotopology) | DSI(Initial Left Neighbourhood) | Results |
|---|---|---|---|
| Mouth | 0 | 0 | Complete Instability |
| Stomach | 0 | 0.25 | Partial Instability |
| Liver | 0 | 0 | Complete Instability |
| Small Intestine | 0 | 0.25 | Partial Instability |
| Liver and Pancreas | 0.67 | 0 | Complete Instability |
| Stomach and Small Intestine | 0 | 0.4 | Partial Instability |
| Whole Digestive System | 1 | 1 | Full stability |

## 6.Conclusion

This study demonstrates that nanotopology is an effective mathematical framework for evaluating the stability of complex biological networks like the human digestive system. By applying the **Digestive Stability Index (DSI)**, This study quantifies how individual organs often exhibit complete instability (**DSI=0)** when viewed in isolation, whereas the **Whole Digestive System** consistently achieves full stability(**DSI=1).**Furthermore, comparing the two frameworks reveals that the **Initial Left Neighbourhood**structure provides a more precise measurement of partial stability for subsystems like the stomach and small intestine. Ultimately, this nanotopological approach confirms that biological stability is a systemic property that emerges only through the interdependent interaction of all constituent organs.Moreover, this method can be applied to any real-life scenario. The same approach can be extended to various fields such as the medical field, academic-related fields, marketing, business sectors, and others.

## References:

1.Brindha.S, Sathishmohan.P, & Rajendran.V, (2019),Attribute reduction in Polycystic Ovary Syndrome via nano topology using basis.,*Advances and Applications in Mathematical Sciences*, *18*, 1487–1497.

2.Jayalakshmi. A, & Janaki.C, (2017),A new class of sets in nano topological spaces with application in medical diagnosis. *International Journal of Applied Engineering Research*, *12*(16), 5894–5899.

3.Lellis Thivagar. M, & Richard.C, (2013), On nano forms of weakly open sets,*International Journal of Mathematics and Statistics Invention*, *1*(1), 31–37.

4.Maheswari.R, & Kathiroli.K, (2023), Diagnosing hyperthyroidism in women using the concepts of nano topology,*International Journal of Research Publication and Reviews*, *4*(10), 3083–3092.

5.Maheswari.R & Lavanya.P,(2023),Elimination of attributes in peptic ulcers in human being via nano topology,*International Journal of Research Publication and Reviews*, *4*(10), 3072–3082.

6.Maheswari.R, Nesha.S & Malathi.R, (2025),Evaluating the symptoms of Chronic Obstructive Pulmonary Diseases (COPD) in youngsters by using initial right nano topological space,*International Research Journal of Education and Technology*, *7*(2).

7.Maheswari.R & Rekha.K,(2024),$\mathfrak{N}_{\bar{\mathcal{S}}}\mathfrak{D}_{\mathfrak{L}}\mathfrak{N}\ell$ and$\mathfrak{N}_{\overline{\mathcal{QS}}}\mathfrak{D}_{\mathfrak{L}}\mathfrak{N}\ell$ spaces in $\mathfrak{D}_{\mathfrak{L}}$- nano topological space,*Indian Journal of Science and Technology*, *17*(43), 4552–4557.

8.Maheswari.R, & Rekha.K,(2024),Initial grill nano topological spaces with initial grill nano generalized closed sets,*Jnanabha*, *54*(2), 76–82.

9.Maheswari.R & Rekha.K(2025),The extensions of Hausdorffness and regularity in $\mathfrak{D}_{\mathfrak{R}}$-nano topological space,*International Journal of Research in Engineering & Science*, *9*(4), 82–90.



 

10.Maheswari.R & Rekha.K (2025). The assessment of various normal spaces in $\mathfrak{D}_{\mathfrak{R}}$-nano topological space,*International Journal of Novel Research and Development*, *10*(8), 718–721.

11.Pawlak.Z,(1982). Rough sets. *International Journal of Computer & Information Sciences*, *11*(5), 341–356.

12.Rekha.K & Maheswari.R, (2025),Nutrition analysis using DL-Nano topological spaces and DR-nano topological spaces through attribute reduction method. *International Journal of Science and Research*, *14*(8), 576–580.

13.Rekha. K& Maheswari.R,(2025),An utilization of DL-Nano topological spaces in assessing the key determinants of stress. *International Research Journal of Education and Technology*, *8*(8), 554–562.

14.Rekha.K& Maheswari.R,(2025),The generalization of Hausdorffness and regularity in $\mathfrak{D}_{\mathfrak{L}}$-nano topological space. *Indian Journal of Natural Sciences*, *16*(91), 99468–99473.

15.A.A.Nasef, M. Shokry and M. Hassan, Some Graph Operators and Its Applications, Asian Journal of Mathematics and Computer Research, 25(6), 340-352, 2018

16.M.Shokry and R.Emad Aly, Topological Properties on Graph via Medical Application in Human Heart, International Journal of Applied Mathematics,.15( 1), 2013,1-10.



 